BOSWELL-BÈTA

James Boswell Exam
VWO Mathematics C
Solution Key

Date:	Practice exam 1
Time:	3 hours
Number of questions:	6
Number of subquestions:	20
Number of supplements:	1
Total score:	68

Subject-specific marking rules and guidelines

1. For each error or mistake in calculation a single point will be subtracted from the maximum score that can be obtained for that particular part of the question.
2. If a required explanation, deduction or calculation has been omitted or has been stated incorrectly 0 points will be awarded, unless otherwise stated in the solution key. This is also the case for answers obtained by the use of a graphic calculator. Answers obtained by the graphic calculator should indicate how the graphic calculator has been used to obtain the answer. Candidates must make sure they mention formulas applied or provide lists and calculation methods used in their answers.
3. If a notational error has been made, but the error can be seen to have no influence on the final result, no points will be deducted from the total score. If, however, it is not possible to determine that there is no influence on the final result a point will be deducted from the final score.
4. A particular mistake in the answer to a particular exam question will lead to a deduction of points only once, unless the question is substantially simplified by the mistake and/or when the solution key specifies otherwise.
5. A repeated mistake made in the answer to different exam questions will lead to a deduction of points each time such a mistake has been made, unless the solution key specifies otherwise.
6. If only one example, reason, explication, explanation or any other type of answer is required and more than one has been given, only the first answer given will be graded. If more than one example, reason, explication, explanation or any other type of answer is required, only the first answers are graded, up to and including the number of answers specified by the exam question.
7. If the candidate fails to give a required unit in the answer to a question a single point will be subtracted from the total score, unless the unit has been specified in the exam question.
8. If during intermediate steps results are rounded, resulting in an answer different from one in which non-rounded intermediate results are used, one point will be subtracted from the total score. Rounded intermediate results may, however, be noted down
Exceptions to this rule are those cases in which the context of the question requires the rounding of intermediate results. The <u>maximum</u> number of points deducted from the total score due to rounding errors is 2 for the entire exam.

Examples for the exceptions to rule 8.
Rounding off intermediate results can be forced by the context if, for example
- The amount of money for a single good has to be rounded to two decimals;
- The number of persons, things, etc. in a concrete situation (i.e. not for an average or expected value) has to be rounded to the nearest integer.

A required level of accuracy can be forced by the context if, for example
- The answer would not be distinguishable from a trivial answer. This can occur with the rounding of growth factors or probabilities to 0 or 1. A probability of $\left(\frac{1}{6}\right)^5$ may be rounded to 0.0001 but not to 0.000.

The forced rounding up or down of answers can occur, for example
- If the exam question specifies a minimum or maximum amount. (For example, if the question is: 'What is the minimal distance an athlete has to jump to gain a certain number of points in a contest?')

The above examples by no means exhaust all possible cases.

Question 1: Electric cars

a	*Method 1:*	
	For each car Chris has $5 \cdot 2 \cdot 3$ $(= 30)$ possibilities	2
	Chris can place $30 \cdot 30 = 900$ different orders that satisfy his requirement	1
	Method 2:	
	For the paint colour there are 5^2 $(= 25)$ possibilities For the rim size there are 2^2 $(= 4)$ possibilities For the interior colour there are 3^2 $(= 9)$ possibilities	2
	Chris can place $25 \cdot 4 \cdot 9 = 900$ different orders that satisfy his requirement	1
b	$v = 100$ gives $P = 0.00002 \cdot 100^3 - 0.0025 \cdot 100^2 + 0.24 \cdot 100 = 19$ (kilowatts) $v = 130$ gives $P = 0.00002 \cdot 130^3 - 0.0025 \cdot 130^2 + 0.24 \cdot 130 = 32.89$ (kilowatts)	1
	$\frac{32.89 - 19}{19} \cdot 100\% \approx 73.1\%$	2
c	$\frac{20}{1.36} = 14.70 \ldots$	1
	Insight that the equation $0.00002 \cdot v^3 - 0.0025 \cdot v^2 + 0.24 \cdot v = 14.70 \ldots$ has to be solved	1
	Describing how this equation can be solved ○ $Y_1 = 0.00002x^3 - 0.0025x^2 + 0.24x$ and $Y_2 = 14.70 \ldots$ ○ Option intersect gives $x = 85.34 \ldots$	1
	The answer: 85.3 km per hour	1
d	$E = \dfrac{100 \cdot (0.00002v^3 - 0.0025v^2 + 0.24v)}{v}$	1
	Describing how the minimum can be calculated ○ $Y_1 = \frac{100 \cdot (0.00002x^3 - 0.0025x^2 + 0.24x)}{x}$ ○ Option minimum gives ($x = 62.5$ and) $y = 16.18 \ldots$	1
	The minimal energy consumption is (approximately) 16.2 (kilowatt-hours per 100 km)	1

Question 2: Dominos

a	$\text{P(no double)} = \frac{21}{28} \cdot \frac{20}{27} \cdot \frac{19}{26} \cdot \frac{18}{25} \cdot \frac{17}{24} \cdot \frac{16}{23} \cdot \frac{15}{22} \left(= \frac{21 \text{ nPr } 7}{28 \text{ nPr } 7} \right)$	2
	The answer: 0.098	1
	Award at most one point to the answer $\text{P(no double)} = \left(\frac{21}{28}\right)^7 \approx 0.133$	
b	There are 21 dominos left on the table (the domino with a double 6 and 20 others)	
	$\text{P(Lucas may start)} = \binom{7}{1} \cdot \frac{1}{21} \cdot \frac{20}{20} \cdot \frac{19}{19} \cdot \frac{18}{18} \cdot \frac{17}{17} \cdot \frac{16}{16} \cdot \frac{15}{15}$	2
	The answer: $\frac{1}{3}$ or 0.333	1
c	If the first domino is a double, 6 other dominos can be connected to it The candidate is allowed to indicate this on the supplement to the exam	1
	$\text{P(first domino is double and the second one can be connected)} = \frac{7}{28} \cdot \frac{6}{27}$	1
	If the first domino is not a double, 12 other dominos can be connected to it The candidate is allowed to indicate this on the supplement to the exam	1
	$\text{P(first domino is not a double and the second domino can be connected)} = \frac{21}{28} \cdot \frac{12}{27}$	1
	$\text{P(two dominos can be connected)} = \frac{7}{28} \cdot \frac{6}{27} + \frac{21}{28} \cdot \frac{12}{27} \approx 0.389$ (or with a different number of decimals)	1

Question 3: A sequence

a	The common difference $v = \left(\frac{\Delta u}{\Delta n} =\right) \frac{4096 - 10\,000}{4} = -1476$	1
	$u_0 = 10\,000 + 2 \cdot 1476 = 12\,952$	1
	$u_n = u_{n-1} - 1476$ with $u_0 = 12\,952$	1
b	The common ratio $r = \left(\frac{4096}{10\,000}\right)^{1/4} = 0.8$	2
	$u_0 = \frac{10\,000}{0.8^2} = 15\,625$	1

Question 4: Trees

a	Prove that H and D are not directly proportional *Method 1:* $\frac{3.15}{0.06} = 52.5$ and $\frac{12.62}{0.45} \approx 28$ These ratios are not equal, so H and D are not directly proportional *Method 2:* When H becomes $\frac{12.62}{3.15} \approx 4$ times as large, D becomes $\frac{0.45}{0.06} = 7.5$ times as large $4 \neq 7.5$, so H and D are not directly proportional *Method 3:* Suppose $D = a \cdot H$ $H = 3.15$ and $D = 0.06$ gives $a = \frac{0.06}{3.15} \approx 0.019$, so $D \approx 0.019 \cdot H$ $H = 12.62$ then yields $D \approx 0.019 \cdot 12.62 \approx 0.24$ (metres) $0.24 \neq 0.45$, so H and D are not directly proportional	2
	Prove that H and D are not inversely proportional *Method 1:* When H increases, so does D. This means that H and D are not inversely proportional (because if they would be, then as H increases, D should *decrease*) *Method 2:* $3.15 \cdot 0.06 = 0.189$ and $12.62 \cdot 0.45 = 5.679$ These products are not equal, so H and D are not inversely proportional	1
b	Insight that the equation $0.01 \cdot H^{1.5} = 7.7$ needs to be solved	1

	Describing how the equation can be solved
	• algebraically:
	    ○ $0.01 \cdot H^{1.5} = 7.7$
	    ○ $H^{1.5} = 770$
	    ○ $H = 770^{\frac{1}{1.5}}$ $(= 84.0 \ldots)$
	• graphic-numerically:
	    ○ $Y_1 = 0.01 \cdot x^{1.5}$ and $Y_2 = 7.7$
	    ○ Option intersect gives $x = 84.0 \ldots$
		1

	The answer: 84 (metres)	1
c	$0.01 \cdot H^{1.5} = D$ $H^{1.5} = \dfrac{1}{0.01} \cdot D \; (= 100 \cdot D)$	1
	$H = (100 \cdot D)^{\frac{1}{1.5}}$	1
	$H = 100^{\frac{1}{1.5}} \cdot D^{\frac{1}{1.5}}$	1
	$H \approx 21.54 \cdot D^{0.67}$(, so $p \approx 21.54$ and $q \approx 0.67$)	1
d	$H = 10^{1.5} \approx 31.6$ (metre)	1
	$D = 10^{-0.5} \approx 0.3$ (metre)	2

Question 5: Three teachers

a	If Carlos would be a maths teacher, then he would tell the truth	1
	This leads to a contradiction, because he claims that he is a chemistry teacher So Carlos cannot be a maths teacher	2
b	*Method 1:*	
	Carlos is not a maths teacher, so neither is Alice (because she is lying)	1
	So Bob is the maths teacher	1
	This means that Bob is telling the truth: Carlos teaches physics	1
	Then Alice teaches chemistry	1
	Method 2:	
	Alice is lying, so Bob or Carlos *must* tell the truth	
	If Carlos is telling the truth (and teaches chemistry), then Alice and Bob are lying. This is impossible, because one of them must be the maths teacher and thus tell the truth.	1
	This means that Bob is telling the truth: Carlos teaches physics	1
	Alice is lying, so she teaches chemistry	1
	Then Bob is the maths teacher	1

Question 6: Coffee

a	The growth factor per hour is equal to $\left(1 - \frac{12}{100} =\right) 0.88$	
	Insight that the equation $0.88^t = \frac{1}{2}$ needs to be solved	1

	Describing how the equation can be solved
	• algebraically:
	    ○ $0.88^t = \frac{1}{2}$
	    ○ $t = \log_{0.88}\left(\frac{1}{2}\right)$ $(= 5.42 \ldots)$
	• graphic-numerically:
	    ○ $Y_1 = 0.88^x$ and $Y_2 = \frac{1}{2}$
	    ○ Option intersect gives $x = 5.42 \ldots$
		1

	The answer: 5.4 hours	1
b	$40 \cdot 0.932^t = C$ $0.932^t = \frac{1}{40} \cdot C$ $(= 0.025 \cdot C)$	1
	$t = \log_{0.932}\left(\frac{1}{40} \cdot C\right)$	1
	$C = 8$ gives $t = \log_{0.932}\left(\frac{1}{40} \cdot 8\right) = 22.854 \ldots$	1
	$0.854 \ldots \cdot 60 = 51.2 \ldots$ so at 07.51 (the next day)	2
c	$X =$ the amount of coffee dispensed by the machine (in mL). $X \sim Norm(120.0, 3.5)$	
	$invNorm(0.75, 120.0, 3.5)$ $(= 122.36 \ldots)$	1
	The answer: (at least) 122.4 (mL)	1
d	$Y =$ the number of cups that contain more than 125 mL	
	$\mathrm{P}(X > 125.0) = normalcdf(125.0, 10^{99}, 120, 3.5)$ $(= 0.0765 \ldots)$	1
	Insight that Y is binomially distributed with $n = 5$ and $p = 0.0765 \ldots$	1
	$\mathrm{P}(Y = 2) = binompdf(5, 0.0765 \ldots, 2)$ (or $\mathrm{P}(Y = 2) = \binom{5}{2} \cdot 0.0765 \ldots^2 \cdot (1 - 0.0765 \ldots)^3$)	1
	The answer: 0.046	1
e	$S =$ the total amount of coffee in the five cups (in mL)	
	Insight that $\mu_S = 5 \cdot 120.0 = 600.0$ and $\sigma_S = \sqrt{5} \cdot 3.5$ $(= 7.826 \ldots)$	2
	$\mathrm{P}(595.0 < S < 605.0) = normalcdf\left(595.0, 605.0, 600.0, \sqrt{5} \cdot 3.5\right)$ $(= 0.4770 \ldots)$	1
	The answer: 0.477	1